

9.3 Monte Carlo Simulation to Model Deterministic Behavior

File: *Area.nb*

Introduction to Computational Science: Modeling and Simulation for the Sciences
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Ex. Area Under a Curve

Monte Carlo simulation to find the area under a curve that is above the x -axis from $x = a$ to $x = b$

For example, find area under $f(x) = x^2$
between $x = 2$ and $x = 3$.

General concept:

Throw darts at rectangle containing area
Count percentage that hit under curve
Take that percentage of area of rectangle

- Clear out old values for f and x and then define f .
- Find the value exactly using a definite integral.
- Plot f from 2 to 3 in *Mathematica*.
- Plot f in *Mathematica* showing the origin and the desired region.
- We generate random floating point y values between 0 and 9 and corresponding random floating point x values between 2 and 3. For each pair, we determine if the random y value is below the function at the random x value. If so, we consider that the "dart" with coordinates of the random x and y values hit below the curve.

Generate a table, *dartTbl*, of ten 0s and 1s so that an entry is 1 if a random y value is less than the function at a random x value and that the element is 0 otherwise.

- Calculate the fraction (*fractionUnder*) of darts that hit under the curve.

- Calculate the area (*rectArea*) of the rectangular dartboard.

- Determine an estimate for the area under the curve between 2 and 3.

- We get a better estimate if we use more "darts." Copy the statements that assign values to *dartTbl*, *fractionUnder*, *rectArea*, and *area* into one cell. Revise the statement that assigns a value to *dartTbl* to generate a table of 1000 values but not to display the result. Execute the subsequent commands to obtain a better estimate of the area under the curve.

We obtain a better result if we perform this process a number of times and calculate the average (mean) area. The standard deviation gives an indication of how good the result is.

- Generate a 100-by-100 table of 0s and 1s, where each row consists of one simulation of throwing 100 darts.

- Calculate the fraction (*fractionUnder*) of darts that hit under the curve for each row (simulation), placing the results in a list.

- Load the package *Statistics`DescriptiveStatistics`*.

- Find the mean and standard deviation of the results for our simulations, and express the answers as floating point numbers.

Repeat the process for the transpose

- Take the transpose of *dartTbls*.

- Calculate the fraction (*fractionUnder*) of darts that hit under the curve for each row (simulation), placing the results in a list.

- Find the mean and standard deviation of our simulation results, and express the answers as floating point numbers.

Estimate the area using values from the entire table

- Flatten the table *dartTbls*.

- Calculate the fraction (*fractionUnder*) of darts that hit under the curve for the entire table (simulation), and find the area as a floating point number.